CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2013 series

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

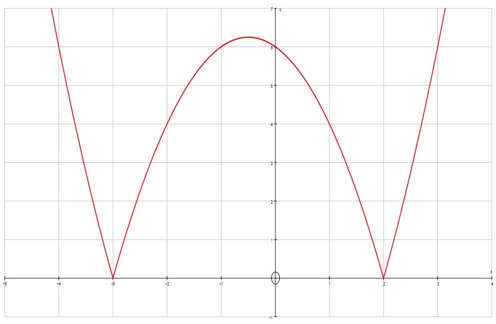
- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

1	$a = 3,\ b = 2,\ c = 1$	B1, B1, B1 [3]	B1 for each
2	Using $b^2 - 4ac$, $9 = 4(k+1)^2$ $4k^2 + 8k - 5 = 0$	M1 DM1	M1 for any use of $b^2 - 4ac$ DM1 for solution of their quadratic in k
	$k = -\frac{5}{2}, \left(\frac{1}{2}\right)$	A1	A1 for critical value(s), $\frac{1}{2}$ not necessary
	To be below the x-axis $k < -\frac{5}{2}$	A1 [4]	A1 for $k < -\frac{5}{2}$ only
	Or: $\frac{dy}{dx} = 2(k+1)x - 3$ when $\frac{dy}{dx} = 0$, $x = \frac{3}{2(k+1)}$ $\therefore y = (k+1)\frac{9}{4(k+1)^2} - \frac{9}{2(k+1)} + (k+1)$ To lie under the x-axis, $y < 0$		
	$\therefore (k+1)\frac{9}{4(k+1)^2} - \frac{9}{2(k+1)} + (k+1) < 0$ leading to $9 = 4(k+1)^2$ or equivalent then as for previous method	M1	M1 for a complete method to this point.

3 $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}+\frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	M1	M1 for dealing with the fractions, denominator must be correct, be generous with numerator
$=\frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$		
$=\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$	DM1	M1 for expansion and use of $\cos^2\theta+\sin^2\theta=1$
$=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$	DM1	M1 for attempt to factorise
$=2\sec\theta$	A1 [4]	A1 for obtaining final answer correctly
Alternative solution: $\sec\theta+\tan\theta+\frac{1}{\sec\theta+\tan\theta}$		
$=\frac{(\sec\theta+\tan\theta)^2+1}{\sec\theta+\tan\theta}$	M1	M1 for dealing with the fractions
$=\frac{\sec^2\theta+2\sec\theta\tan\theta+\tan^2\theta+1}{\sec\theta+\tan\theta}$		
$=\frac{2\sec^2\theta+2\sec\theta\tan\theta}{\sec\theta+\tan\theta}$	DM1	M1 for expansion and use of $\tan^2\theta+1=\sec^2\theta$
$=\frac{2\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}$	DM1	DM1 for attempt to factorise
$=2\sec\theta$	A1	A1 for obtaining final answer correctly
4 **(i)** n (A) = 3	B1 [1]	If elements listed for **(i)**, then they must be correct elements to get B1 leading to n (A) = 3. If they are not listed and correct answer given then B1.
(ii) n (B) = 4	B1 [1]	If elements listed for **(ii)**, then they must be correct elements leading to n (B) = 4 to get B1. If they are not listed and correct answer given then B1.
(iii) $A \cup B$ = {60°, 240°, 300, 420°, 600°}	√B1 [1]	Follow through on any sets listed in **(i)** and **(ii)**. Do not allow any repetitions.
(iv) $A \cap B$ = {60°, 420°}	√B1 [1]	Follow through on any sets listed in **(i)** and **(ii)**.

5 **(i)** $9x - \frac{1}{3}\cos 3x\,(+c)$	B1, B1, B1 [3]	B1 for $9x$, B1 for $\frac{1}{3}$ or $\cos 3x$ B1 for $-\frac{1}{3}\cos 3x$ Condone omission of $+ c$
(ii) $\left[9x - \frac{1}{3}\cos 3x\right]_{\frac{\pi}{9}}^{\pi}$		
$= \left(9\pi - \frac{1}{3}\cos 3\pi\right) - \left(\pi - \frac{1}{3}\cos\frac{\pi}{3}\right)$	M1	M1 for correct use of limits in their answer to **(i)**
$= 8\pi + \frac{1}{2}$	A1, A1 [3]	A1 for each term
6 $f\left(\frac{1}{2}\right) = \frac{a}{8} + 1 + \frac{b}{2} - 2$	M1	M1 for substitution of $x = \frac{1}{2}$ into $f(x)$
leading to $a + 4b - 8 = 0$	A1	A1 for correct equation in any form
$f(2) = 2f(-1)$	M1	M1 for attempt to substitute $x = 2$ **or** $x = -1$ into $f(x)$ **and** use $f(2) = \pm 2f(-1)$ or $2f(2) = \pm f(-1)$
$8a + 16 + 2b - 2 = 2(-a + 4 - b - 2)$	A1	A1 for a correct equation in any form
leading to $10a + 4b + 10 = 0$ or equivalent $\therefore a = -2,\ b = \frac{5}{2}$	DM1 A1 [6]	DM1 (on both previous M marks) for attempt to solve simultaneous equations to obtain either a or b A1 for both correct

7 **(a)** **(i)** 360	B1 [1]	
(ii) 120	B1 [1]	
(b) **(i)** 924	B1 [1]	
(ii) 28	B1 [1]	
(iii) $924 - \left({}^8C_3 \times {}^4C_3\right) - \left({}^8C_2 \times {}^4C_4\right)$ (i.e. 924 – 3M 3W – 2M 4W)	M1	M1 for 3 terms, at least 2 of which must be correct in terms of C notation or evaluated.
924 – 224 – 28	A1	A1 for any pair (must be evaluated)
= 672	A1 [3]	A1 for final answer
Or: 4M 2W ${}^8C_4 \times {}^4C_2$ = 420	M1	M1 for 3 terms, at least 2 of which must be correct in terms of C notation or evaluated.
5M 1W ${}^8C_5 \times {}^4C_1$ = 224	A1	A1 for any pair (must be evaluated)
6M ${}^8C_6$ = 28		
Total = 672	A1	A1 for final answer
8 **(i)** 	B1	B1 for correct shape
	B1	B1 for (–3, 0) or –3 seen on graph
	B1	B1 for (2, 0) or 2 seen on graph
	B1 [4]	B1 for (0, 6) or 6 seen on graph or in a table
(ii) $\left(-\frac{1}{2}, \frac{25}{4}\right)$	B1, B1 [2]	B1 for each
(iii) $k > \frac{25}{4}$ or $\frac{25}{4} < k\ (\leq 14)$	B1 [1]	

9 **(a)** $12x^2\ln(2x+1)+4x^3\left(\frac{2}{2x+1}\right)$	M1 A2, 1, 0 [3]	M1 for differentiation of a correct product –1 for each error
(b) **(i)** $\frac{dy}{dx}=\frac{(x+2)^{\frac{1}{2}}2-2x(x+2)^{-\frac{1}{2}}\frac{1}{2}}{x+2}$	M1, A1	M1 for differentiation of a quotient involving $(x+2)^{\frac{1}{2}}$ A1 all correct unsimplified
$=\frac{(x+2)^{-\frac{1}{2}}}{(x+2)}(2(x+2)-x)$	DM1	DM1 for attempt to simplify
$=\frac{x+4}{(x+2)^{\frac{3}{2}}}$	A1 [4]	A1 for correct simplification to obtain the given answer
Or: $\frac{dy}{dx}=2x\left(-\frac{1}{2}\right)(x+2)^{-\frac{3}{2}}+(x+2)^{-\frac{1}{2}}(2)$	M1, A1	M1 for differentiation of a product involving $(x+2)^{-\frac{1}{2}}$
$=(x+2)^{-\frac{3}{2}}(2(x+2)-x)$		A1 all correct unsimplified
$=\frac{x+4}{(x+2)^{\frac{3}{2}}}$	DM1 A1	DM1 for attempt to simplify A1 for correct simplification to obtain the given answer
(ii) $\frac{10x}{\sqrt{x+2}}\ (+c)$	M1,A1 [2]	M1 for $\frac{1}{5}\times\frac{2x}{\sqrt{x+2}}$ or $5\times\frac{2x}{\sqrt{x+2}}$ A1 correct only, allow unsimplified. Condone omission of $+c$
(iii) $\left[\frac{10x}{\sqrt{x+2}}\right]_2^7=\frac{70}{3}-\frac{20}{2}$	M1	M1 for correct application of limits in their answer to **(b)(ii)**
$=\frac{40}{3}$	A1 [2]	

10 (i) $\sqrt{20}$ or 4.47	B1 [1]	
(ii) Grad $AB = \frac{1}{2}$, $\perp$ grad = –2	M1	M1 for attempt at a perp gradient
$\perp$ line $y - 4 = -2(x - 1)$	M1, A1	M1 for attempt at straight line equation, must be perpendicular and passing through *B*.
$(y = -2x + 6)$	[3]	A1 allow unsimplified
(iii) Coords of *C* (x, y) and $BC^2 = 20$ $(x-1)^2 + (y-4)^2 = 20$ **or** Coords of *C* (x, y) and $AC^2 = 40$	M1	M1 for attempt to obtain relationship using an appropriate length and the point (1, 4) or (–3, 2)
$(x+3)^2 + (y-2)^2 = 40$	A1	A1 for a correct equation
Need intersection with $y = -2x + 6$,	DM1	DM1 for attempt to solve with $y = -2x + 6$ and obtain a quadratic equation in terms of one variable only
leads to $5x^2 - 10x - 15 = 0$ **or** $5y^2 - 40y - = 0$		
giving $x = 3, -1$ and $y = 0, 8$	DM1 A1, A1 [6]	M1 for attempt to solve quadratic A1 for each 'pair'
Or, using vector approach:		
$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$	B1	May be implied
$\overrightarrow{OC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$	M1 A1, A1	M1 for correct approach A1 for each element correct
$\overrightarrow{OC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$	A1,A1	A1 for each element correct

11 (a) (i) $\begin{pmatrix} 4 & 3 \\ 4 & 3 \end{pmatrix}$	B1 [1]	
(ii) $\mathbf{A}^2 = \begin{pmatrix} 16 & 9 \\ 12 & 13 \end{pmatrix}$	B1, B1 [2]	B1 for any 2 correct elements B1 for all correct
(iii) **B** is the inverse matrix of $\mathbf{A}^2$ $= \frac{1}{100}\begin{pmatrix} 13 & -9 \\ -12 & 16 \end{pmatrix}$	√B1, √B1 [2]	Follow through on their $\mathbf{A}^2$
(b) det **C** $= x(x-1) - (-1)(x^2 - x + 1)$ $= 2x^2 - 2x + 1$	M1 A1	M1 for attempt to obtain det **C** A1 for this correct quadratic expression from a correct det **C**
$b^2 - 4ac < 0$, $4 - 8 < 0$	DM1	DM1 for use of discriminant or attempt to solve using the formula, or attempt to complete the square in order to show there are no real roots.
No real solutions (so det **C** $\neq 0$)	A1 [4]	A1 for correct reasoning or statement that there are no real roots.

12 (a) (i) $f(-10) = 299$, $f(8) = 191$ Min point at $(0, -1)$ or when $y = -1$ $\therefore$ range $-1 \le y \le 299$	M1 B1 A1 [3]	M1 for substitution of either $x = -10$ or $x = 8$, may be seen on diagram B1 May be implied from final answer, may be seen on diagram Must have $\le$ for A1, do not allow x
(ii) $x \ge 0$ or equivalent	B1 [1]	Allow any domain which will make f a one-one function Assume upper and lower bound when necessary.
(b) (i) $g^{-1}(x) = \ln\left(\frac{x+2}{4}\right)$ or $\dfrac{\lg\left(\frac{x+2}{4}\right)}{\lg e}$	M1 A1 [2]	M1 for complete method to find the form inverse function, must involve ln or lg if appropriate. May still be in terms of y. A1 must be in terms of x
(ii) $gh(x) = g(\ln 5x)$ $= 4e^{\ln 5x} - 2$ $20x - 2 = 18$, $x = 1$	M1 A1 A1 [3]	M1 for correct order A1 for correct expression $4e^{\ln 5x} - 2$ A1 for correct solution from correct working
Or $h(x) = g^{-1}(18)$ $\ln 5x = \ln 5$ leading to $x = 1$	M1 A1 A1	M1 for correct order A1 for correct equation A1 for correct solution **from correct working**